Model for the calculation of the radar cross section of wake vortices of take-off and landing airplanes

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Abstract—**A new model has been developed for the calculation of the radar cross section and the power backscattered by wake vortices of take-off and landing airplanes in order to better understand the physical mechanisms involved, to compare with the measurements that will be performed in X-band by Thales during the SESAR P12.2.2 project and to prepare inputs for testing a Wake Vortex Advisory System. The model contains a first part that simulates the evolution of the vortex in stratified atmosphere, using simplified 2D fluid mechanics equations and uses the pressure, temperature and humidity for the calculation of the difference between the dielectric permittivity of the clear air and of the wake vortex. The second part calculates the power backscattered by the dielectric permittivity gradients in the vortex. The input parameters describe the atmosphere as well as the airplane of interest. Simulation results have been compared with results published in the literature, when all the input parameters are available.**

Keywords- Radar cross section;wake vortices;air trafic security

I. INTRODUCTION

Real-time sensing of wake vortices at take-off and landing is a key issue for the implementation of new air traffic management systems for reduced aircraft separation in favourable weather conditions and for increased air safety. Wake vortices consist in two counter rotating swirling flows generated at the tips of the wings and the outer flaps. Even if they are only visible in certain conditions of temperature and humidity, they are always present and could be responsible for induced roll movement of the trailing aircraft. To avoid damages to the trailing aircraft, the International Civil Aviation Organization (ICAO) has provided separation criteria based on a fixed distance or time separation between two aircrafts. To increase traffic, it is envisaged to monitor the wake vortices in order to work with a variable separation depending of the meteorological parameters responsible for the vortex decay.

In the framework of SESAR P12.2.2 project, a model is developed at UCL for the calculation of the radar cross-section of the wake vortices in clear air, for a wide frequency range. The radar signal simulator is planned to be used in a Wake Vortex Scenario Generator that will further be used for testing a Wake Vortex Advisory System. The radar simulator is divided in two parts: the wake vortex simulator that calculates the time evolution of the pressure, temperature and humidity in the wake vortex, in function of the airplane type and velocity, the tropospheric parameters, for a stable atmosphere. This part of the software also calculates the dielectric permittivity of the humid air. The second part of the software calculates the radar cross-section of the vortex and the power received at the radar antenna. This necessitates the calculation of an oscillating integral. Two methods have been compared and the most accurate has been chosen.

The radar cross section calculation has been validated by comparison with results available in the literature, when all the input parameters used in the software are given. The radar parameters are still to be implemented (antenna pattern, pulse shape, averaging, etc.) to deliver the inputs for the Wake Vortex Advisory System.

II. CAPABILITY OF RADAR DETECTION OF WAKE VORTICES

Lidar sensors are currently used for wake vortex measurements but their performances are degraded in rainy or foggy conditions [1].

Since the eighties, radar measurements of wake vortices showed an evidence of detection for frequencies going from VHF to X band [2][3][4][5]; these campaigns are described in [6]. Recent measurements with a 35 GHz pulsed Doppler radar [7] or with high resolution 94 GHz radar [8] report the possibility to detect aircraft wake vortices in fog and light rain, with a reasonable power and distances lower than 2 km (500 m for the W band). Barbaresco [9][10] has shown that trials performed with an X-band Doppler radar, are able to continuously detect and characterize the strength as well as to profile wake vortices to a range of 2000m with the radar used.

If the ability for the radar to detect aircraft wake vortices in all weather conditions is now well established, the scattering characteristics of the wake vortices are not well modeled. In order to optimize the radar parameters and to simulate the power backscattered by wake vortices of various airplanes in various atmospheric conditions, a simulator is being developed for radar backscattering in clear air. Other authors are working on the X-band radar backscattering of wake vortices in rain and fog [11][8].

III. SIMULATION OF THE DIELECTRIC PERMITTIVITY OF WAKE VORTICES

A. Dielectric permittivity of the wake vortex

The backscattering of radiowaves depends on the relative dielectric permittivity of the wake vortices. In order to calculate the backscattering, it is necessary to determine the relative permittivity of the wake vortices with respect to the clear air permittivity

$$\varepsilon_r = \varepsilon_{r,a} + \Delta\varepsilon_r \tag{1}$$

$\varepsilon_{r,a}$ being the relative permittivity of ambient air and ε_r the permittivity of the wake vortex. The square root of the permittivity is the refractive index n of air, representing the ambient air when used with “a” subscript. Due to the low value of the permittivity of air, refractivity is used

$$N = (n-1)10^6 \tag{2}$$

and the relative permittivity variation $\Delta\varepsilon_r$ can be linearized as

$$\Delta\varepsilon_r \approx 2(n - n_a) \tag{3}$$

The accuracy of the linearization has been checked for extreme values of temperature and specific humidity. The advantage of the linearization is that it enables to separate the effects of the various constituents: dry air, water vapor and carbon dioxide.

Thayer [12] has proposed a semi-empirical formula for clear air for frequencies lower than 20 GHz:

$$N = 0.776\frac{p_d}{T} + 1.33\frac{p_{CO2}}{T} + 0.648\frac{e}{T} + 3.77610^3\frac{e}{T^2} \tag{4}$$

where p_d is the dry air partial pressure [Pa], T is the absolute temperature [K], p_{CO2} is the partial pressure of CO_2 [Pa], e is the water vapor partial pressure [Pa] and the compressibility factors of dry air and water are assumed to be unity. Two mechanisms causing refractive index gradients are generally considered, as presented in [6][10]:

- The radial density gradient in the vortex cores having a lower pressure and so a lower refractive index than the surrounding medium. This mechanism mainly depends on the airplane type that influences the intensity of the flow
- The transport of the atmospheric fluid in the oval surrounding the vortices. This mechanism transports the air from one place to another, assuming adiabatic compression when the oval descents and the ambient pressure increases.

A third mechanism is linked to the turbo reactor stream that seems to influence the refractive index in the region close to the aircraft due to high temperature. It will however not be considered in this preliminary model because we are only interested in the vortices a few wingspan away from the airplane, as explained in the next section.

The model presented here does not use the first two mechanisms separately but combines the effects of water vapor and pressure in the calculation of the dielectric permittivity. The combination of equations (2), (3) and (4), neglecting the dipolar and carbon dioxide terms, becomes

$$\Delta\varepsilon_r \approx 210^{-6}\left[0.776\left(\frac{p_d}{T} - \frac{p_{da}}{T_a}\right) + 0.648\left(\frac{e}{T}\left(\frac{5827}{T}\right) - \frac{e_a}{T_a}\left(\frac{5827}{T_a}\right)\right)\right] \tag{5}$$

where the “a” subscript indicates the ambient air.

B. Simplified calculation of the pressure, temperature and humidity in the wake vortex

The general parameters used for the description of the vortices are the following

$$V_0 \cong \frac{\Gamma_0}{2\pi b_0}\ ;\ b_0 = \frac{\pi B}{4}\ ;\ \Gamma_0 = \frac{Mg}{\rho V b_0}$$

$$t_0 = \frac{b_0}{V_0}\ ;\ \tau = \frac{t}{t_0}$$

where Γ_0 is the root velocity, V_0 the descent velocity of the vortex pair, b_0 the initial vortex spacing, B the wingspan, V and M are the aircraft velocity and mass, ρ is the air density, g the gravity constant, t_0 is the characteristic time.

The first step is the simulation of the wake vortex pressure, temperature and humidity that will enable the calculation of space and time repartition of the dielectric permittivity of the vortex, as given by equation (5). Vortices are generated at the inner and outer flaps, at the horizontal stabilizer as well as at the wing tips of the aircraft. At a distance of a few wingspans behind the aircraft, they recombine to produce the two main vortices separated by a distance b_0 and having a circulation Γ_0.

The incompressible Navier-Stokes equation with the Boussinesq approximation describes the movement. The water vapor concentration obeys to the convection-diffusion equation, being considered as a passive tracer that does not influence the velocity, pressure and temperature fields but is “carried away” by the fluid. The last equation used is the energy equation.

2D pseudo-spectral numerical methods are used for the resolution of the differential equations, so that all the fields have to be periodical and this is not straightforward with stratification of the ambient atmosphere. This imposes the use of some mathematical artifacts to represent periodic pressure, water vapor, velocity and potential temperature. The system of differential equations is then solved by a Runge-Kutta method of order 3. The model for wake vortices necessitates high Reynolds numbers and a very dense mesh. Due to computer limitations, the small scales are not modeled. The introduction of a hyper-viscous term in the equations enables the dissipation of the high wavenumbers, in a way similar to the one used in Large Eddy Simulations. So, the dissipation term only affects the highest wavenumbers.

This combined model has been compared to the models proposed by Shariff and shows a good agreement. Fig. 1 shows the comparison between the reduced pressure in the vortex core, calculated with the model presented here and the model of Shariff [6]. The parameters are described at the beginning of this section. Fig. 2 presents the variation between the dielectric permittivity in the oval surrounding the vortices and the ambient air, in function of the horizontal position. The accuracy is good so that the two mechanisms are considered as well modeled.

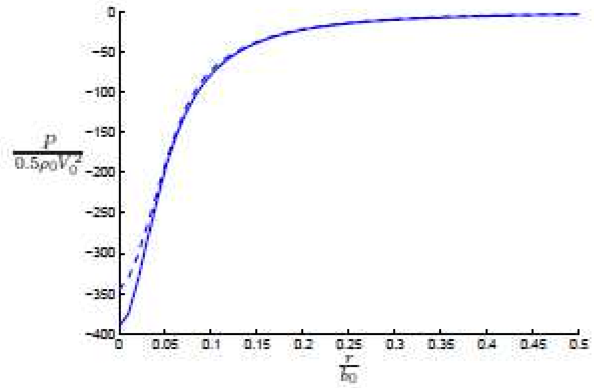


Figure1. Reduced pressure in the vortex core (- -) simulation by the model presented and (—) by the method of Shariff [6]

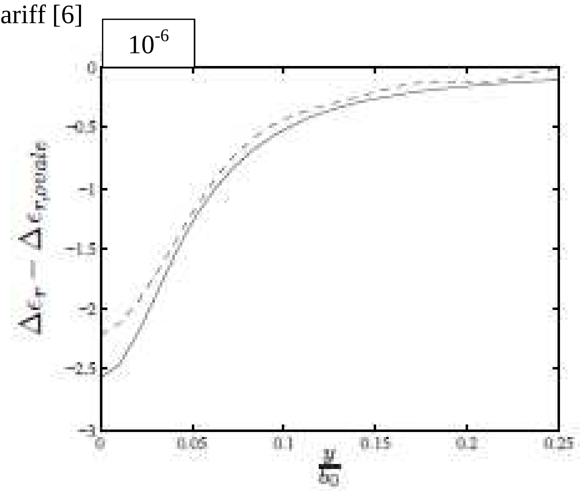


Figure 2. Variation of the dielectric permittivity relative to ambient air in function of the horizontal position, for τ=2, simulation by the model presented and (—) by the method of Shariff [6]

IV. CALCULATION OF THE RADAR BACKSCATTERING OF THE WAKE VORTEX

The electromagnetic signal transmitted by the radar is backscattered by the presence of gradients in the refractivity of the troposphere. The size of the volume observed by the radar is defined by the beam width in the transverse plane and by the pulse length and shape in the direction of propagation. The signal received by the radar is the combination of the scattered contributions of that large volume. The resolution of the generated turbulent volume is determined by the thermodynamic simulation and is around 50 cm. Better resolution necessitates too large storage capacity and becomes also expensive in computing time if we remember that simulation will be performed for various airplanes and radars types.

The first step is the calculation of the radar cross section of the wake vortex and the power received by the radar which is the combination of scattered contributions of inhomogeneities of various sizes.

The electric and magnetic field refracted by a volume of dielectric objects are calculated using the Hertz vector

$$\Pi(\overline{x_r}) = \frac{1}{4\pi}\int_V (\varepsilon_r(\bar{x}) - 1)\bar{E}(\bar{x})\frac{e^{-j k_0 |\overline{x_r} - \bar{x}|}}{|\overline{x_r} - \bar{x}|}dV$$

where $\overline{x_r}$ indicates the position of the receiver and $\bar{x}$ indicates the local coordinates of the vortex. The Born approximation is used and the total electric field in the vortex $\bar{E}(\bar{x})$ is replaced by the incident field. The power received at the radar is given by

$$P_r(\overline{x_r}) = \frac{\omega\varepsilon_0 k_0^3 A^2}{32\pi^2|\overline{x_r}|^4}\,|I|^2$$

$$I = \int_V (\varepsilon_r(\bar{x}) - 1) f(x,y,z) e^{-2jk_0\xi} dV$$

$$\sigma_r = \frac{k_0^2}{4\pi}|I|^2$$

Where ξ is the distance between the receiver and the volume element dV, k_0 is the incident wavenumber, f(x,y,z) is the radiation pattern, A is the amplitude of the incident field and σ_r is the radar cross section. I is an oscillating integral because the exponent is rapidly oscillating with position of the volume dV: the wavenumber is very high, especially in X-band which is our frequency of interest. The integration method proposed by Li [13][14] replaces the integral by the resolution of a system of differential equations. The method has been compared to the one of Shariff and Way and is more precise.

V. EXAMPLE OF CALCULATION OF RADAR CROSS SECTIONS

Since the simulation of the wake vortex evolution is 2D, slices of dielectric permittivity are produced versus time, giving the evolution of the wake vortices. The input parameters of the simulation are given in Table I.

TABLE I. INPUT PARAMETERS OF THE SIMULATION

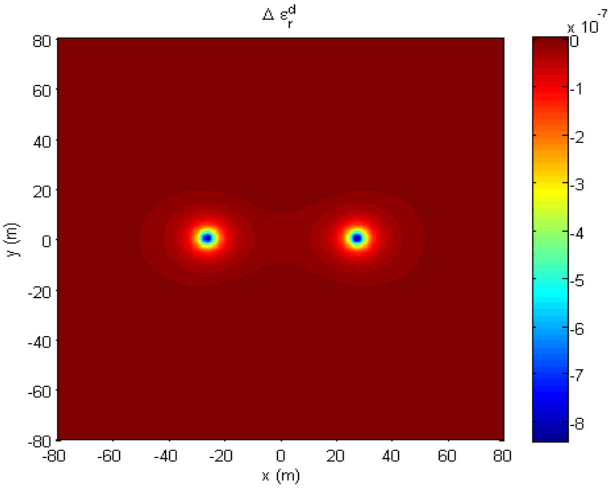
Parameters	**Values**
Airplane mass	M=250000 Kg
Wingspan	B=68m
Airplane velocity	V =133m/s
Ambient pressure	$p_a = 100000\,pa$
Ambient absolute temperature	$T_a = 288\,K$
Water vapor content gradient	$m_q = -8\times10^{-8}$ Kg/mKg

The evolution of the two parts of the dielectric constant: the dry air part and the water vapor part are represented in Fig. 3. The "2D slice" is taken after 5 seconds evolution time. The evolution of the water vapor part is clearly visible in function of time and is given after 20 seconds evolution time in Fig. 4. The value of $\Delta\varepsilon_r$ increases for increasing evolution time. The calculation of the radar cross section has been performed for the geometry shown in Fig. 5. For this first calculation, the elevation angle is assumed to be 90 degrees. The radar cross section is calculated for a “slice” of 1m in the direction of the axis of the vortex. This contribution has to be further integrated on the beamwidth of the radar antenna.

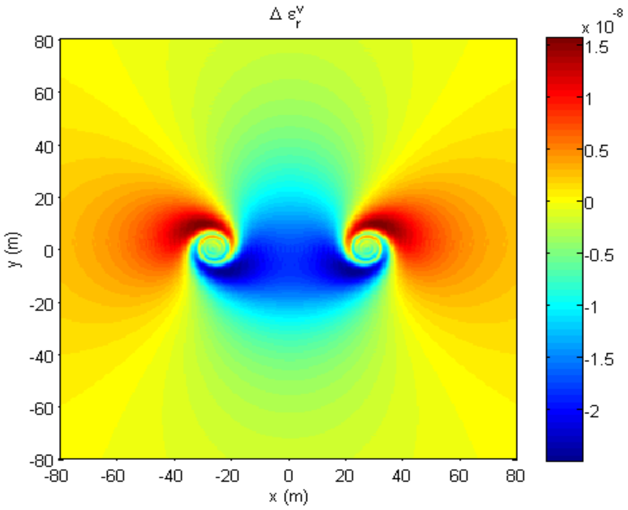
In order to compare the radar cross section with results from the literature, the radar cross section has been calculated for a “slice” of 1m in the low frequency range and the results are in agreement with the results obtained by Li [15]. It has however to be stressed that the effects of the scales smaller than the resolution are not taken into account. A further extension using statistical characteristics for the smaller scales could be envisaged in a further development.

VI. CONCLUSION

A model has been developed for the calculation of the pressure, temperature and humidity in airplane wake vortices, in order to calculate the dielectric permittivity inside the vortices and to evaluate the radar cross section and the power backscattered to the radar. This model will be used for a parametric study of the power received by radar. This parametric study is needed for the optimization of the radar parameters and for a better understanding of the physical mechanisms responsible for the signal received by the radar. It will also serve to the development of a look up table that will be used used for testing a Wake Vortex Advisory System.



(a)



(b)

Figure 3. Relative dielectric constant variation due to density (a) and water vapor (b) variation 5s after roll up

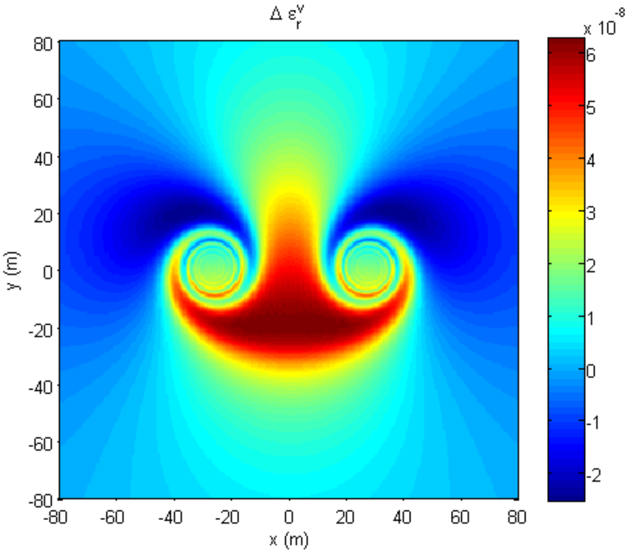


Figure 4. Relative dielectric constant variation due to water vapor 20s after roll up

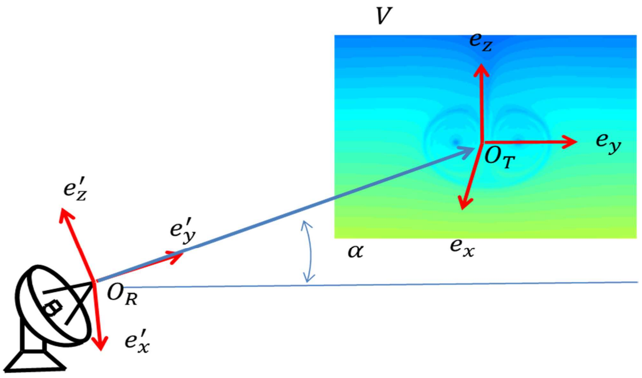


Figure 5. Geometry of the radar for the calculation of the radar cross section

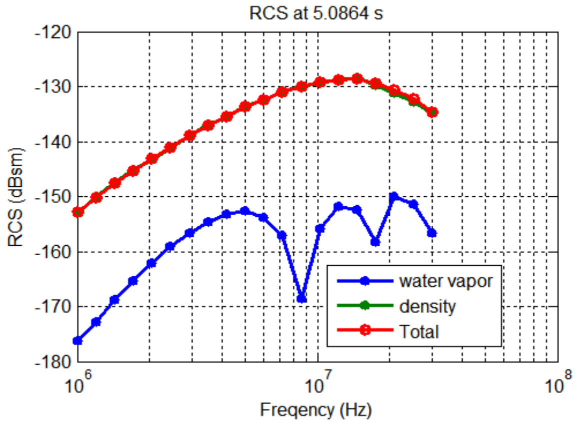


Figure 6. Radar cross section versus frequency in the lowest frequency band

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