

Wake Encounter Severity Assessment Based on Validated Aerodynamic Interaction Models

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Wake encounter severity criteria based on validated models are of great importance for any wake vortex related severity assessment. The aerodynamic interaction model “strip method” describes the vortex-induced aircraft reaction. The model quality is validated with wake encounter flight test data. Model shortcomings are improved with dedicated refinements. A simplified hazard area approach is developed employing validated simulation models. With one simple criterion, the roll control ratio, safe and undisturbed flight operations can be ensured. A limit value for manually flown (non fly-by-wire) aircraft is derived from piloted trials. With the simplified hazard area prediction method, this can be universally applied for wake vortex advisory systems like DLR’s wake vortex prediction and monitoring system.

Nomenclature

<p>a = acceleration</p> <p>α = angle of attack</p> <p>b = wing span</p> <p>β = angle of sideslip</p> <p>C_l = rolling moment coefficient</p> <p>$C_{L\alpha}$ = lift gradient</p> <p>δ = control device deflection</p> <p>Δ = difference</p> <p>Γ = circulation/vortex strength</p> <p>H = altitude</p> <p>r = yaw rate</p> <p>r_c = vortex core radius</p> <p>p = roll rate</p> <p>ρ = air density</p> <p>q = pitch rate</p> <p>t = time</p> <p>V = velocity</p> <p>W = weight</p> <p>subscripts</p> <p>0 = initial value</p> <p>a = aileron</p> <p>HTP = horizontal tail plane</p> <p>i = index</p> <p>L = leader aircraft, lift</p> <p>max = maximum</p> <p>t = tangential</p> <p>WV = wake vortex</p> <p>x,y,z = longitudinal, lateral, vertical coordinates</p>	<p>abbreviations</p> <p>A/C aircraft</p> <p>AIM Aerodynamic Interaction Model</p> <p>ATM Air Traffic Management</p> <p>ATTAS Advanced Technologies Testing Aircraft System</p> <p>cg center of gravity</p> <p>DoF degrees of freedom</p> <p>DLR German Aerospace Center <i>(Deutsches Zentrum für Luft- und Raumfahrt)</i></p> <p>D2P deterministic two phase model</p> <p>FAA Federal Aviation Administration</p> <p>ICAO International Civil Aviation Organization</p> <p>IFS in-flight simulation</p> <p>ILS instrument landing system</p> <p>IMC instrument meteorological conditions</p> <p>MTOW maximum takeoff weight</p> <p>P2P probabilistic two phase model</p> <p>RCR roll control ratio</p> <p>SESAR Single European Sky ATM Research</p> <p>SHA simplified hazard area</p> <p>SHAPE simplified hazard area prediction</p> <p>VMC visual meteorological conditions</p> <p>WSVBS wake vortex prediction and monitoring system <i>(Wirbelschleppen Vorhersage- und Beobachtungssystem)</i></p> <p>ZFB Center of Flight Simulation Berlin <i>(Zentrum für Flugsimulation Berlin)</i></p>
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I. Introduction

ASSESSING the severity of wake vortex encounters is of high relevance for both designing any wake vortex advisory and assistance systems and conducting the corresponding safety cases. For the design of advisory and assistance systems the severity assessment should not be of high complexity and should not require lots of computation time and input data in order to be applied in an operational environment. This means that a certain amount of simplification is required. On the other hand for the safety evaluation of any new wake vortex regulations, operational concepts and advisory or assistance systems a detailed analysis is required. In any case the underlying models and simulations have to be validated with measurement data in order to build confidence.

It is agreed that currently no commonly accepted criteria or metrics for wake vortex encounter severity assessment are available^{1,2,3} although considerable work has been conducted in the last decades⁴ and recently^{5,6}. Currently this topic plays a role in the WakeNet3-Europe task group “safety assessment” and in the activities RECAT* (FAA/Eurocontrol) and SESAR.

This paper presents a simplified approach for wake vortex encounter severity assessment (simplified hazard areas – SHA). The severity assessment is based on aerodynamic interaction models, whose model quality is discussed and validated with test data.

II. Wake Vortex Encounter Modeling

The essential components for wake encounter analysis are the description of the vortex generation, transport and decay, as well as the representation of the wake vortex induced velocity distribution. Another important element is the modeling of the interaction between vortex flow disturbance and encountering aircraft. This section describes the selected models.

A. Wake Vortex Modeling

The initial vortex strength (circulation) Γ_0 according to the KUTTA-JOUKOWSKY theorem for an aircraft (leader "L") in level flight with the weight W_L , wing span b_L , and airspeed V_L is

$$\Gamma_0 = \frac{W_L}{\rho V_L b_L \frac{\pi}{4}} \quad (1)$$

where $\pi/4$ applies for elliptical lift distribution and ρ is the air density. In the literature the initial vortex core radius is specified between 1% and 5% of the generator wing span⁷. According to measurements from flight tests the initial core radius r_{c0} is identified to be 3.5% of the generator wing span^{8,9}

$$r_{c0} = 0.035 b_L \quad (2)$$

Vortex evolution (decay and transport) are modeled by the probabilistic/deterministic two phase model (P2P/D2P) depending on the atmospheric conditions^{10,11}. The core radius r_c is growing with increasing vortex age. Different simulations revealed that the core diameter has no significant effect on the upset of encountering aircraft^{12,13}. Parameter variation showed that for hazard considerations a smaller core radius for a given circulation is a conservative approach¹⁴. Therefore a constant core radius is used.

$$r_c = r_{c0} \quad (3)$$

The wake vortex induced velocities are calculated by superimposing two single vortices, using the analytical tangential velocity (V_t) model of BURNHAM-HALLOCK¹⁵ (based on ROSENHEAD¹⁶), which yields good results for wake vortex encounters^{8,9}.

$$V_t = \frac{\Gamma_L}{2\pi} \cdot \frac{r}{r_c^2 + r^2} \quad (4)$$

B. Aerodynamic Interaction Model Validated with Flight Tests

The effects on aircraft aerodynamics due to individual angle of attack variations induced by spatial local atmospheric flow disturbances need to be considered by an Aerodynamic Interaction Model (AIM). An accepted and easy to apply encounter model is the *Strip Model*^{18,19}. It is based on lifting line theory and describes the additional aerodynamic forces and moments acting on an aircraft in a spatial wind field, e.g. wake turbulence. The important

* RECAT - Re-categorization of the Wake Turbulence Separation Minima, Eurocontrol and FAA joint effort

aerodynamic forces generating surfaces of an aircraft (wing, horizontal and vertical tail) are segmented into strips, Figure II-1. Well proven numbers of strips for a medium size aircraft to cover the effects of the wake vortex phenomenon are 16 for the wing, 8 for the horizontal tail plane and 4 for the vertical tail. At the 25% chord location of each strip the additional angles of attack (wing, horizontal tail) and angles of sideslip (vertical tail) produced by the local flow disturbance are computed. Applying the respective local lift gradient $C_{L\alpha}$ an additional lift portion is determined for each strip. These local lift increments are weighted in span direction elliptically and then summarized to get the resulting forces. Additionally, the corresponding moments of all strips are computed from their lift increments and their individual lever arms with respect to the aircraft's cg. The summary of the moment increments results in the overall moments. No drag effects are considered initially by the strip model, describing wake effects in 5 degrees of freedom.

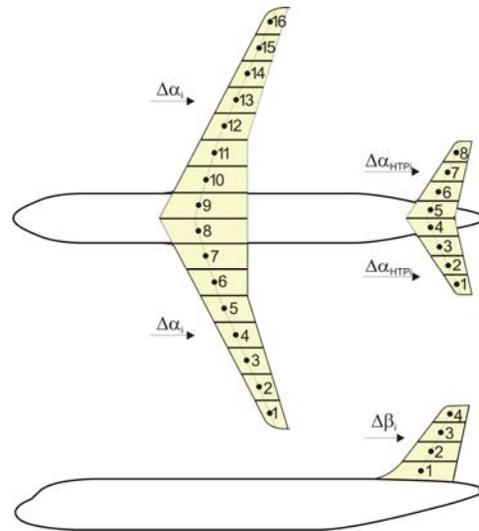


Figure II-1. Strip Model.

The validation process of the AIM with flight test data is illustrated in Fig. II-2. The complete aerodynamic model computes the sum of forces and moments of (a) the basic aircraft aerodynamic model and (b) the aerodynamic interaction model, which provides Δ -forces and -moments due to wake influence. The simulation is driven by the flight test measured control inputs (elevator, aileron, rudder etc.). The model outputs are compared to the corresponding measured data, which are typically linear and rotational accelerations, rotational rates, altitude, and velocity. It is self-evident that the basic aerodynamic model has to be of high quality to make sure that the calculated error (“*model accuracy*”) does not originate from this model. The required quality can only be achieved by tuning the basic aerodynamic model with parameter identification techniques in an a-priori step using suitable flight test data that are recorded far away from any wake influence. In such flight tests the a/c eigenmodes should be excited²⁰ adequately to identify the respective parameters.

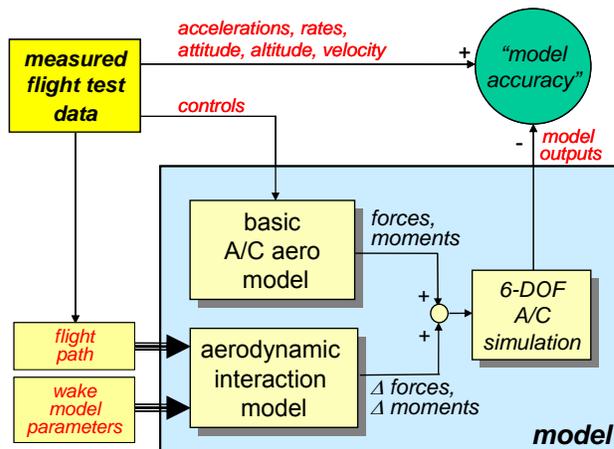


Figure II-2. Method to validate wake encounter models from flight test data

In addition the exact knowledge of the wake vortex model parameters (strength and position) for each encounter is required. These model parameters are determined also in an a-priori step. Using flight test data of the encounter aircraft, its flight path can be reconstructed and the corresponding flow angles α and β can be computed precisely without any local flow effects from wake influence taken into account. The differences between these inertial flow angles and the measured ones are used to determine the wake characteristics.

Finally, the aerodynamic interaction model is fed with the reconstructed flight path and the aircraft's Euler angles. This “driven mode” stabilizes the wake encounter simulation and proved to be essential, as wake induced forces and moments are very sensitive to small flight path inaccuracies.

The accuracy is assessed by computing the standard deviations of the error between model outputs and the corresponding flight test data. The maximum errors are also observed. Each degree of freedom is considered separately.

Applying the method of Fig. II-2, a validation example is shown in Fig. II-3: a lateral fly-through of the twin engine turboprop Do128 aircraft (MTOW 4 t) into the wake of DLR's twin engine jet VFW 614 ATTAS test aircraft (MTOW 21 t). This fly-through is a typical validation example out of more than 50 encounters conducted in the European project S-Wake^{18,21}. Typical model outputs (red lines) in all 6 DoF are compared to the corresponding flight test data (black lines).

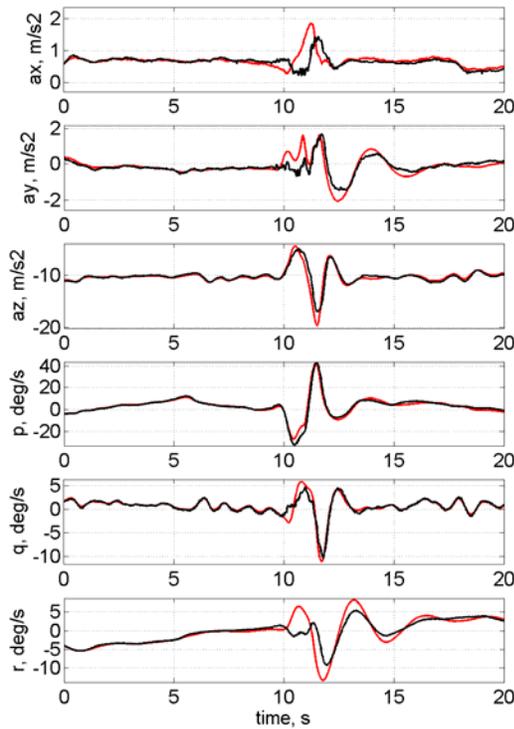


Figure II-3. Do128 lateral wake fly-through: simulation model output (—) compared to flight test data (—)

drag. Applying corresponding lever arms, the drag increments were also added to the yawing moment.

A further model extension was introduced to consider fuselage effects. For this empirical model the fuselage is divided typically into 20 strips (Fig. II-4), computing a wake induced local sideslip angle at each fuselage strip. Using a suitable fuselage strip derivative, the summation of the strip increments gives a lateral fuselage force, and, considering the corresponding lever arms, a corresponding fuselage yawing moment.

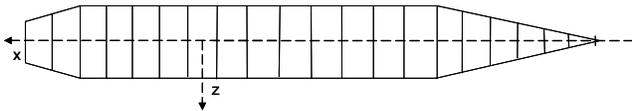


Figure II-4. Strip model fuselage effect modelling

ATTAS wake providing high quality flight test data. The result: both derivatives were identified to be about 0.8. Looking at the standard deviations of the errors between the flight test data and the simulation output, a considerable model improvement for all analysed encounters is realized: about 51% in the yaw (r) and 48% in the longitudinal axis (ax) for all 23 encounters. Through coupling effects, improvements also in the roll axis p (16%), in the lateral axis ay (10%) and the vertical axis az (11%) are achieved.

Figure II-5 shows the time histories of the encounter case already discussed in Figure II-3 but now applying the described model extensions. Despite some discrepancies in the lateral motion, a considerable improvement in the longitudinal axis (ax) and the yawing motion (r) is seen. The initial opposite model reaction in the yawing motion is now largely eliminated. However, one constraint is evident: no general formulation was found for the semi-empirical drag derivatives. Suitable values can be determined from flight test data applying the method described in this paper. If those flight test data are not available, the value of 0.8 may be used, but it should be noted that the validity of this number has still to be proven for other aircraft.

Looking at each DoF separately, the model quality can be assessed as follows: the rolling motion ($roll\ rate\ p$) and the vertical motion ($vertical\ acceleration\ az$) during a wake fly-through can be simulated in *high quality*. This can be considered to be an outstanding result for the strip model quality with its widely linear structure, and also a verification of the elaborate validation procedure. Both roll and vertical degree of freedom are the most important inputs into today's wake hazard assessment tools. The pitching motion ($pitch\ rate\ q$) is also simulated in good quality, despite some minor deficiencies at the beginning of the wake encounter. The lateral motion ($acceleration\ ay$) has some minor, but tolerable discrepancies. The longitudinal motion ($acceleration\ ax$) has discrepancies as no drag effects are modelled since this degree of freedom is assumed to be of minor interest. The yawing motion is considered to be of more importance in terms of wake encounter simulation quality. Unfortunately, the simulated model dynamics is at the wake entry *opposite* to what the flight test shows. This is a typical result found in many Do128 encounter validations. Some efforts were undertaken to further improve the model quality, with special analysis in the yawing motion. A correlation was found between the model faults in the longitudinal axis and the yawing motion. Obviously, drag effects have considerable impact on the yaw degree of freedom.

So, the model was extended to consider drag effects. Drag depends nonlinearly on angle of attack. However, nonlinearities cannot be implemented in the strip model *independent* of the basic aircraft aero model, but exactly this independency is the fundamental idea of the strip model in this application. To stick to this approach, a linear formulation with one drag derivative, applied to each single strip, was used to consider wing and tail

The determination of the two additional parameters, the *wing drag derivative* and the *fuselage side force derivative*, was done applying the complete validation procedure (Fig. II-2) in an optimization loop to minimize the discrepancies between model output and flight test data¹⁷. The identification process was performed using flight test data of 23 Do128 encounters into the VFW 614

III. Wake Vortex Encounter Severity Criterion for Simplified Hazard Areas

For operational applications like dynamic wake vortex separations a simple and robust severity criterion is needed. The idea is to define simplified hazard areas (SHA) around wake vortices which have to be avoided in order to ensure safety, passenger comfort and undisturbed flight operations (i.e. no go-arounds) with a conservative approach. The simplified hazard area describes this region outside of which safe and undisturbed operations are ensured^{22,7}. In order to have a simple approach only one parameter is selected for the definition of the SHA. However, the acceptable limit for this parameter which describes the size of the SHA ought to be conservative enough to guarantee that no unacceptable wake vortex encounter takes place considering the complete aircraft reaction.

Especially for approach and landing, which constitutes a capacity bottleneck, encounters with small encounter angles are typical. Here the wake vortex induced rolling moment is the dominating effect for the encountering aircraft^{21,23}. This is especially the case for the outer regions of the wake vortex. These are relevant for the determination of the hazard area dimensions, since the core region has to be avoided in any case (as long as the vortex is not largely decayed), because the stronger effects of the wake vortex in the core region cannot necessarily be compensated for as easily. Therefore, the definition of the simplified hazard areas is based on the induced rolling moment. In order to relate the induced rolling moment to the controllability of the encountering aircraft it is related to the maximum roll control power^{24,31}. This defines the dimensionless wake vortex induced roll control ratio RCR.

$$RCR = |C_{l,WV} / C_l(\delta_{a,max})| \quad (5)$$

This is a widespread measure for wake vortex encounter evaluations²⁵⁻²⁸. Choosing an upper RCR limit defines the SHA, (conservatively) approximated by e.g. a rectangle (Figure III-1). It is important to note that if this RCR limit is sufficiently small, the resulting hazard area covers also all other relevant aspects of aircraft response (like aircraft attitude, accelerations and flight path deviations) affected by a wake vortex. The suitability of this approach has been shown by previous investigations^{22,7}. The following section describes the determination of an appropriate RCR limit.

IV. Severity Criterion for Simplified Hazard Areas

The suitability of a roll control ratio limit for the assessment of wake vortex encounter severity for simplified hazard areas was investigated with pilot-in-the-loop simulator and flight tests with the goal to establish a roll control limit to ensure safe and undisturbed flight operations^{22,7}.

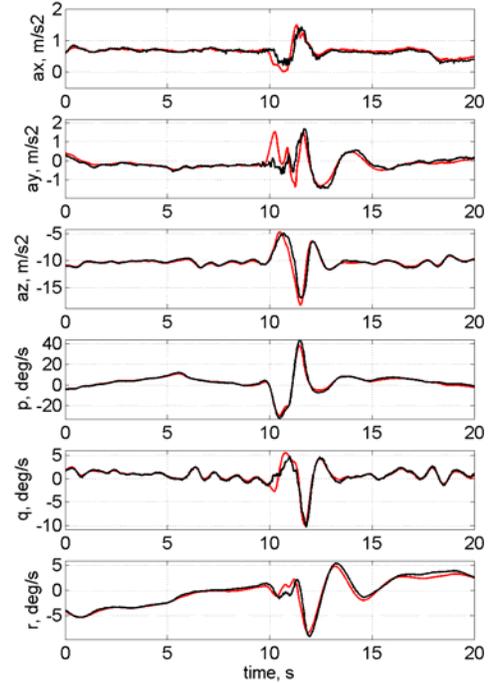


Figure II-5. Do128 lateral wake fly-through: simulation output with wing and fuselage drag modelling (—) compared to flight test data (—)

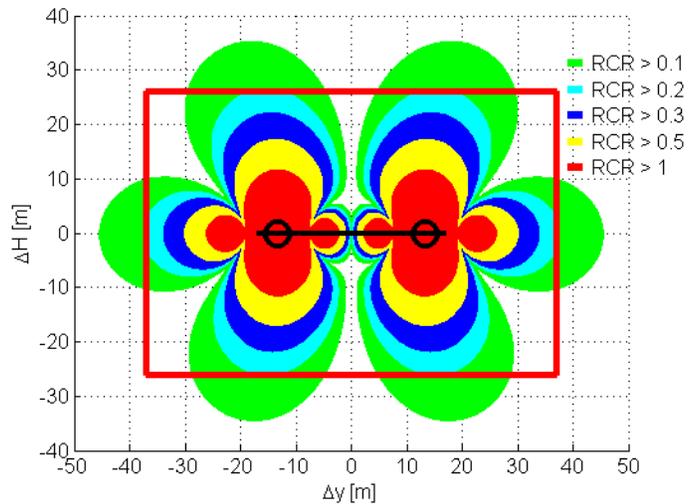


Figure III-1. Wake vortex induced roll control ratio and simplified hazard area (SHA, for RCR = 0.2, vortex age = 50 s, 'light' behind 'medium', no turbulence) in the cross section behind the vortex generating aircraft (with indicated generator wing and vortex cores).

A. Validation Trials

The SHA RCR limit validation trials are piloted aircraft simulations. The aircraft nominal flight path is defined along one of the boundaries of the SHA for specific maximum RCR values (compare red rectangle in Figure III-1). The experiment setup is a typical approach scenario, beginning 6 nm before runway threshold and consisting of an ILS approach and the landing (Fig. IV-1). The pilot task is to track the nominal ILS path following the standard approach procedures to configure the aircraft for landing, including flaps and speed settings, gear operation and go-arounds if applicable. The altitude of the wake encounter and the (relatively small) encounter angles are varied and not known to the pilots. The vortex generating aircraft is in all cases a category 'medium' aircraft (MTOW = 94 t) with a vortex age of $t = 50$ s and a circulation of $\Gamma = 252 \text{ m}^2/\text{s}$.

In addition to the recorded data the pilots give ratings after each approach sequence in four categories: aircraft control, demands on the pilot, aircraft excursions from flight state and path and overall hazard^{30,31}. The rating scale (Fig. IV-2) is graduated into four levels, with a rating of 1 denoting an uncritical case and a 4 denoting an unacceptable one. Ratings of 1-3 are considered acceptable.

Piloted aircraft simulations were conducted using fixed-base, full-flight, and in-flight simulators^{22,7}. In the full-flight simulator of the ZFB (Center of Flight Simulation Berlin – *Zentrum für Flugsimulation Berlin*) 57 analyzable approaches with wake vortex encounters were carried out. The wake vortex induced forces and moments were previously recorded for defined nominal flight tracks and replayed at a defined altitude (“time fixed”) in order to fly exactly along the hazard area boundaries. For small flight path deviations which are generally the case outside of simplified hazard areas the resulting errors can be neglected. The simulated aircraft was a twin engine turboprop (similar to Do 228), MTOW = 6 t, ICAO category ‘light’. Visual conditions were varied (VMC/IMC) and weak background turbulence was present.

Experiments in real flight offer the most realistic simulation environment. This is achieved by means of in-flight simulation (IFS). The DLR research aircraft ATTAS (Advanced Technologies Testing Aircraft System) is specifically designed for this task. The real aircraft acts like the simulated aircraft (in this case the same aircraft type as the real aircraft since the test pilots are experienced with the aircraft type), which encounters the wake vortex. The experimental pilot is flying the simulated aircraft using real controls. These inputs are fed into the onboard computers stimulating the model aircraft which reacts directly to the inputs and to the effects of the virtual wake vortex flow. The resulting model aircraft states and accelerations are fed into the model following control system. The model following controller generates the control commands for the (real) host aircraft which are necessary to make the host aircraft behave like the simulated aircraft. So the flight states of the host aircraft experienced by the experimental pilot are matching the flight states of the simulated aircraft. The feasibility of wake vortex in-flight simulations was previously demonstrated²⁹, exhibiting a good simulation fidelity for an RCR at least up to RCR = 0.5.

The encountering aircraft (ATTAS) type is a VFW 614 (ICAO class 'medium', MTOW = 21 t). Wake vortex encounters are conducted “time fixed” (explanation see above) as well as “space fixed”. In the latter case the vortex pair is positioned near the nominal flight path and vortex-induced forces and moments are calculated online according to the actual aircraft position and attitude. This way the maximum occurring RCR value is not predefined but can be obtained from the recorded data. The flight test preparation is done in DLR’s ATTAS fixed-base system simulator. These results are also taken into account for the hazard analysis.

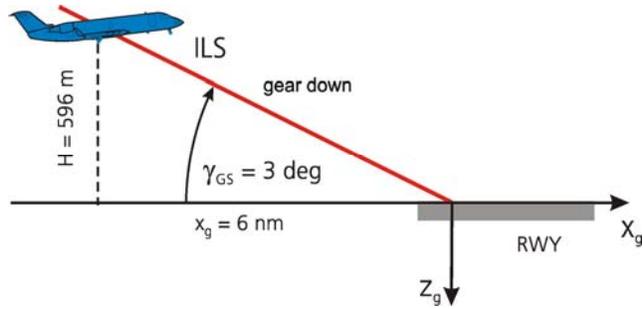


Figure IV-1. Approach scenario (side view).

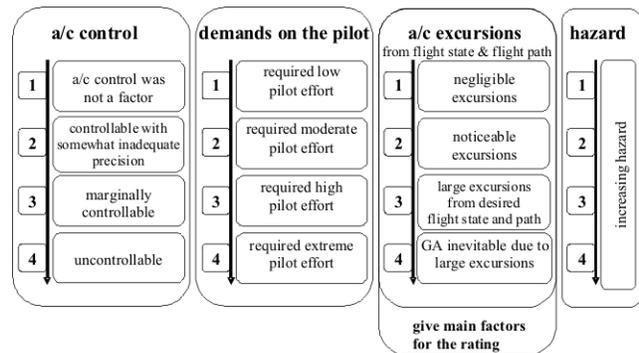


Figure IV-2. Pilot rating scale^{30,31}.

the approach corridor, a save approach is possible for the next aircraft. This way the minimum separation time is derived for a specific position along the approach path. The procedure can be repeated for different windows along the approach corridor, to obtain a minimum separation for the entire approach. This method accounts for the atmospheric conditions and can be executed for any combination of aircraft classes (e.g. “medium” behind “heavy”) and is currently expanded to treat pairings of individual aircraft types. The weather dependent application allows for dynamic separation minima. For a 66 days test period at Frankfurt International Airport the system ran stable and predictions were verified with measurements to be safe³³. Capacity improving concepts could have been applied 75% of the time. Air traffic management simulations show a possible capacity gain of 3%.

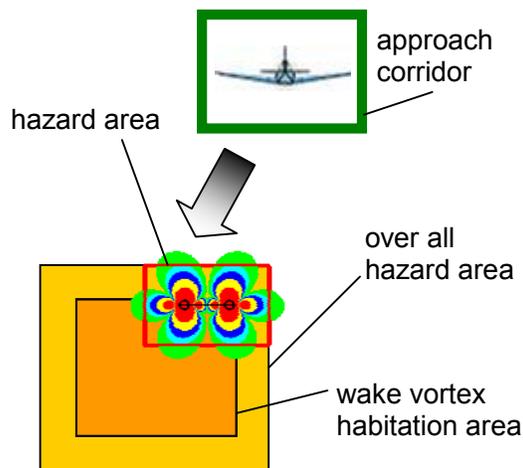


Figure V-1. Approach corridor and hazard areas.

VI. Conclusion

Validated models are essential for wake encounter severity assessment. Specifically the aerodynamic interaction model “strip method” is validated with flight test data from wake vortex encounter in-flight measurements. The unique approach of applying system identification for wake vortex parameter estimation yields good results in order to describe the wake vortex induced aircraft reaction. Further refinements can be successfully applied to the strip method to overcome shortcomings. This provides a solid basis for wake vortex encounter severity considerations. The presented approach for simplified hazard areas (SHA) ensures that unacceptable wake vortex encounters can be avoided. The roll control ratio is a suitable measure for wake vortex encounter severity assessment in the context of wake vortex separation prediction with a conservative limit value of RCR = 0.2 for manually flown (non fly-by-wire) aircraft. The "Simplified Hazard Area Prediction" method (SHAPE), where the aircraft data are parameterized with respect to the maximum takeoff weight, makes the hazard area concept universally applicable to any conventional transport aircraft type. SHAPE represents a major element of DLR’s wake vortex prediction and monitoring system WSVBS for predicting safely reduced dynamic and individual wake vortex related separation minima.

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